

## Econ 252 - Financial Markets

Spring 2011

Professor Robert Shiller

### Midterm Exam #1 - Practice Exam

#### Suggested Solution

#### Part I.

1. Lecture 1 on “Introduction and What This Course Will Do for You and Your Purposes”; assigned reading *The Gospel of Wealth and Other Timely Essays* by Andrew Carnegie.

People successful in business have demonstrated a talent for affairs, and they are morally obligated to use this talent to give away their wealth in a productive way. They should not live ostentatiously, they should retire from business while still in full mental capacity, so that these talents can assure that the wealth is dispensed properly.

2. Lecture 5 on “Insurance, the Archetypal Risk Management Institution, its Opportunities and Vulnerabilities.”

The Health Maintenance Organization Act (HMO Act) encourages businesses to offer a health maintenance organization that incentivizes doctors to watch out for patients’ interests, including preventive medicine, rather than try to justify procedures to make income for themselves.

### 3. Lecture 4 on “Portfolio Diversification and Supporting Financial Institutions.”

For a given set of risky assets, the Efficient Portfolio Frontier represents a special set of portfolios that can be formed out of the given set of risky assets. Namely, each portfolio on the Efficient Portfolio Frontier has minimal standard deviation for a given level of expected return.

The Tangency Portfolio is a special portfolio on the Efficient Portfolio Frontier. It arises from the construction of portfolios consisting of portfolios along the Efficient Portfolio Frontier and of the risk-free asset. Out of all portfolios that can be formed from any portfolio along the Efficient Portfolio Frontier and the risk-free asset, the portfolios consisting of the Tangency Portfolio and the risk-free asset have the highest expected return for a given level of return standard deviation.

An investor with the usual  $\mu$ - $\sigma$  preferences would therefore always choose a portfolio on the line that connects the risk-free asset and the Tangency Portfolio. That means that every investor with the usual  $\mu$ - $\sigma$  preferences will hold the same risky asset, the Tangency Portfolio. Different investors might choose a different weight on the Tangency Portfolio, but no investor will ever choose another risky portfolio along the Efficient Portfolio Frontier to make it part of his or her optimally chosen portfolio. This is the essence of the Mutual Fund Theorem, as the Tangency Portfolio is the only portfolio that needs to be made available to investors, e.g. through a mutual fund.

#### 4. Fabozzi et al., p. 101.

“Monoline insurers guarantee the timely repayment of the bond principal and interest when a bond insurer [sic] defaults on these payments. Monoline insurers are called ‘monoline’ because they provide their insurance services to only one industry, the capital markets.”

(Instead of “bond insurer”, Fabozzi et al. mean to refer to “bond issuer.”)

Examples for the securities that monoline insurers insure include:

- Municipal bonds,
- Collateralized Debt Obligations,
- Collateralized Loan Obligations,
- asset-backed bonds.

## 5. Fabozzi et al., pp. 255-256.

Empirical of the CAPM are motivated by the theoretical implications “that, over the period of time analyzed,

- there is a linear relationship between the average risk premium return on the market and the average risk premium return on a stock or portfolio, and its slope is  $\beta$ , and
- the linear relationship should pass through the origin.

Moreover, according to the CAPM, beta is a complete relative measure of a stock’s risk. Consequently, various alternative risk measure that might be proposed, the most common being the standard deviation of return, should not be significant contributors to the explanation of a stock’s return.”

“The major results of the empirical tests conducted are summarized below:

- The evidence shows a significant positive relationship between realized returns and systematic risk as measured by beta. The average market risk premium estimated is usually less than predicted by the CAPM, however.
- The relationship between risk and return appears to be linear. The studies give no evidence of significant curvature in the risk/return relationship.
- Tests that attempt to discriminate between the effects of systematic and unsystematic risk do not yield definitive results. Both kinds of risk appear to be positively related to security returns, but there is substantial support for the proposition that the relationship between return and unsystematic risk is at least partly spurious - that is, partly a reflection of statistical problems rather than the true nature of capital markets.”

## 6. Guest lecture by David Swensen

College endowment portfolios differ in outcomes primarily in terms of the asset class allocation, not their attempts at timing movements in and out of the stock market.

7. Fabozzi et al., p. 7.

“The factors that have led to the integration of financial markets are

- Deregulation or liberalization of financial markets and the activities of market participants in key financial centers of the world;
- technological advances for monitoring world markets, executing orders, and analyzing financial opportunities;
- increased institutionalization of financial markets.”

8. Lecture 3 on “Technology and Invention in Finance.”

Shareholders cannot be forced to pay for failings of the company (beyond losing their initial investment). Introduced as an 1811 New York State law, it makes investing more lottery-like and fun for risk-averse investors and permits diversification.

9. Lecture 4 on “Portfolio Diversification and Supporting Financial Institutions.”

Average returns on stocks are so high relative to other investments that one wonders why anyone invests in anything else. Ibbotson and also Siegel demonstrate this with long historical data.

10. Guest lecture by David Swensen; Lecture 7 on “Efficient Markets.”

Swensen said it might be misleading, because institutional portfolios involve assets, whose value is only assessed, so standard deviation is spuriously low. Also, investment managers can game the Sharpe ratio, as Goetzmann et al. have shown, by selling or buying tail risk.

**Part II.****Question 1**

(a) Denote the Honest Abe bond by A. It pays \$100 with probability 1. Therefore,

$$E[A] = 1 \cdot 100 = 100.$$

Denote the Bonnie bond by B. It pays \$100 with probability .25+.4=.65 and pays nothing with probability .2+.15=.35. Therefore,

$$E[B] = .65 \cdot 100 + .35 \cdot 0 = 65.$$

Denote the Clyde bond by C. It pays \$100 with probability .25+.2=.45 and pays nothing with probability .4+.15=.55. Therefore,

$$E[C] = .45 \cdot 100 + .55 \cdot 0 = 45.$$

(b) As the Honest Abe bond pays a fixed amount for sure, its variance equals \$0.

The variance of the Bonnie bond equals

$$\text{Var}(B) = E[B^2] - E[B]^2 = .65 \cdot (100)^2 + .35 \cdot (0)^2 - (65)^2 = 2,275.$$

The variance of the Clyde bond equals

$$\text{Var}(C) = E[C^2] - E[C]^2 = .45 \cdot (100)^2 + .55 \cdot (0)^2 - (45)^2 = 2,475.$$

(c) The covariance of the Bonnie bond and the Clyde bond equals

$$\begin{aligned} \text{Cov}(B,C) &= E[B \cdot C] - E[B]E[C] \\ &= .25 \cdot 100 \cdot 100 + .4 \cdot 100 \cdot 0 + .2 \cdot 0 \cdot 100 + .25 \cdot 0 \cdot 0 - 65 \cdot 45 = -425. \end{aligned}$$

(d) The random variable of interest is  $.3A + .6B + .1C$ .

The expected value of this random variable is

$$E[.3A + .6B + .1C] = .3E[A] + .6E[B] + .1E[C] = .3 \cdot 100 + .6 \cdot 65 + .1 \cdot 45 = 73.5.$$

In order to compute the variance of  $.3A + .6B + .1C$ , observe that

$$\text{Var}(.3A + .6B + .1C) = \text{Var}(.6B + .1C),$$

as  $.3A$  is a constant. It follows that

$$\begin{aligned} \text{Var}(.3A + .6B + .1C) &= \text{Var}(.6B + .1C) \\ &= \text{Var}(.6B) + \text{Var}(.1C) + 2 \cdot \text{Cov}(.6B, .1C) \\ &= (.6)^2 \text{Var}(B) + (.1)^2 \text{Var}(C) + 2 \cdot .6 \cdot .1 \cdot \text{Cov}(B, C) \\ &= (.6)^2 \cdot 2,275 + (.1)^2 \cdot 2,475 + 2 \cdot .6 \cdot .1 \cdot (-425) = 792.75. \end{aligned}$$

## Question 2

(a) The return variance satisfies

$$\begin{aligned} \text{Var}(r_p) &= \text{Var}(w \cdot r_A + (1-w) \cdot r_B) \\ &= (w)^2 \cdot \text{Var}(r_A) + (1-w)^2 \cdot \text{Var}(r_B) + 2 \cdot w \cdot (1-w) \cdot \text{Corr}(r_A, r_B) \cdot \text{Std}(r_A) \cdot \text{Std}(r_B). \end{aligned}$$

Using  $w=0.8$  and the information provided for assets A and B, it follows that

$$\begin{aligned} \text{Var}(r_p) &= \text{Var}(w \cdot r_A + (1-w) \cdot r_B) \\ &= (0.8)^2 \cdot (0.42)^2 + (0.2)^2 \cdot (0.17)^2 + 2 \cdot 0.8 \cdot 0.2 \cdot (-0.1) \cdot 0.42 \cdot 0.17 \approx 0.1118. \end{aligned}$$

It follows that the return standard deviation for  $w=0.9$  satisfies

$$\text{Std}(r_p) = \sqrt{\text{Var}(r_p)} = \sqrt{0.1118} \approx 0.3344 = 33.44\%.$$

The expected return satisfies

$$E[r_p] = E[w \cdot r_A + (1-w) \cdot r_B] = w \cdot E[r_A] + (1-w) \cdot E[r_B].$$

Using  $w=0.5$  and the information provided for assets A and B, it follows that

$$E[r_p] = E[w \cdot r_A + (1-w) \cdot r_B] = 0.5 \cdot 0.06 + 0.5 \cdot 0.02 = 0.04 = 4\%.$$

Moreover, using  $w=0.1$  and the information provided for assets A and B, it follows that

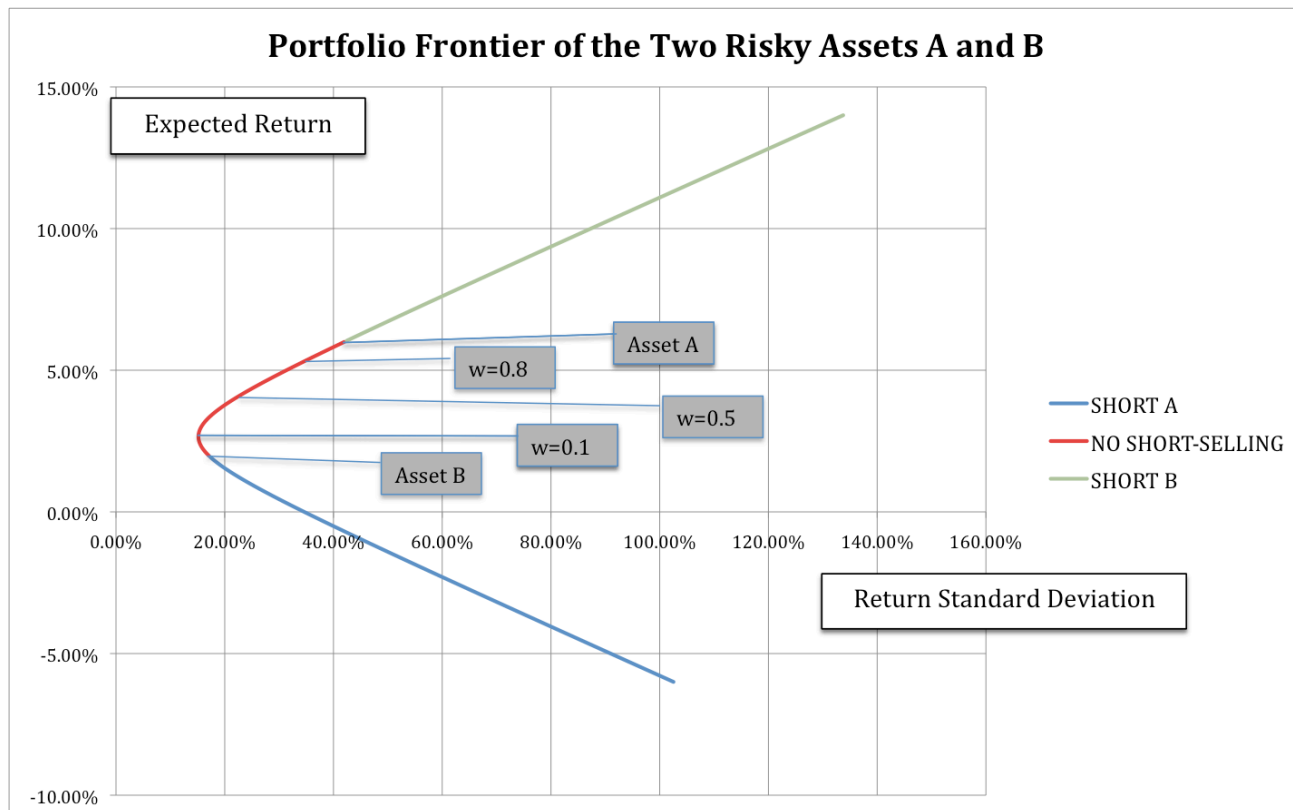
$$E[r_p] = E[w \cdot r_A + (1-w) \cdot r_B] = 0.1 \cdot 0.06 + 0.9 \cdot 0.02 = 0.024 = 2.4\%.$$

In summary, the completed table looks as follows:

Weight	Expected Return	Return Standard Deviation
$w=0.8$	5.2%	33.44%
$w=0.5$	4.0%	21.85%
$w=0.1$	2.4%	15.46%



(b)



(c) The investment in asset A is

$$w \cdot \$50,000 = 0.5 \cdot \$50,000 = \$25,000.$$

The investment in asset B is

$$(1 - w) \cdot \$50,000 = 0.5 \cdot \$50,000 = \$25,000.$$

The return standard deviation in \$ of this portfolio is given by

$$\$50,000 \cdot 21.85\% = \$10,925.$$

(d) The investment in asset A is

$$w \cdot \$50,000 = (-0.5) \cdot \$50,000 = -\$25,000.$$

The investment in asset B is

$$(1 - w) \cdot \$50,000 = 1.5 \cdot \$50,000 = \$75,000.$$

This portfolio position is attained by borrowing \$25,000 worth of asset A and selling this position right away. Then, the proceeds of this sale are used to invest in asset B in addition to the originally available \$50,000.

The variance of a portfolio with weight  $w = -0.5$  on asset A and weight  $w = 1.5$  on asset B is

$$\begin{aligned} \text{Var}[-0.5 \cdot r_A + 1.5 \cdot r_B] &= (-0.5)^2 \text{Var}[r_A] + (1.5)^2 \text{Var}[r_B] + 2 \cdot (-0.5) \cdot 1.5 \cdot \text{Cov}(r_A, r_B) \\ &= (-0.5)^2 \text{Var}[r_A] + (1.5)^2 \text{Var}[r_B] + 2 \cdot (-0.5) \cdot 1.5 \cdot \text{Corr}(r_A, r_B) \cdot \text{Std}(r_A) \cdot \text{Std}(r_B) \\ &= (-0.5)^2 \cdot (0.42)^2 + (1.5)^2 \cdot (0.17)^2 + 2 \cdot (-0.5) \cdot 1.5 \cdot (-0.1) \cdot 0.42 \cdot 0.17 \\ &= 0.1198 \end{aligned}$$

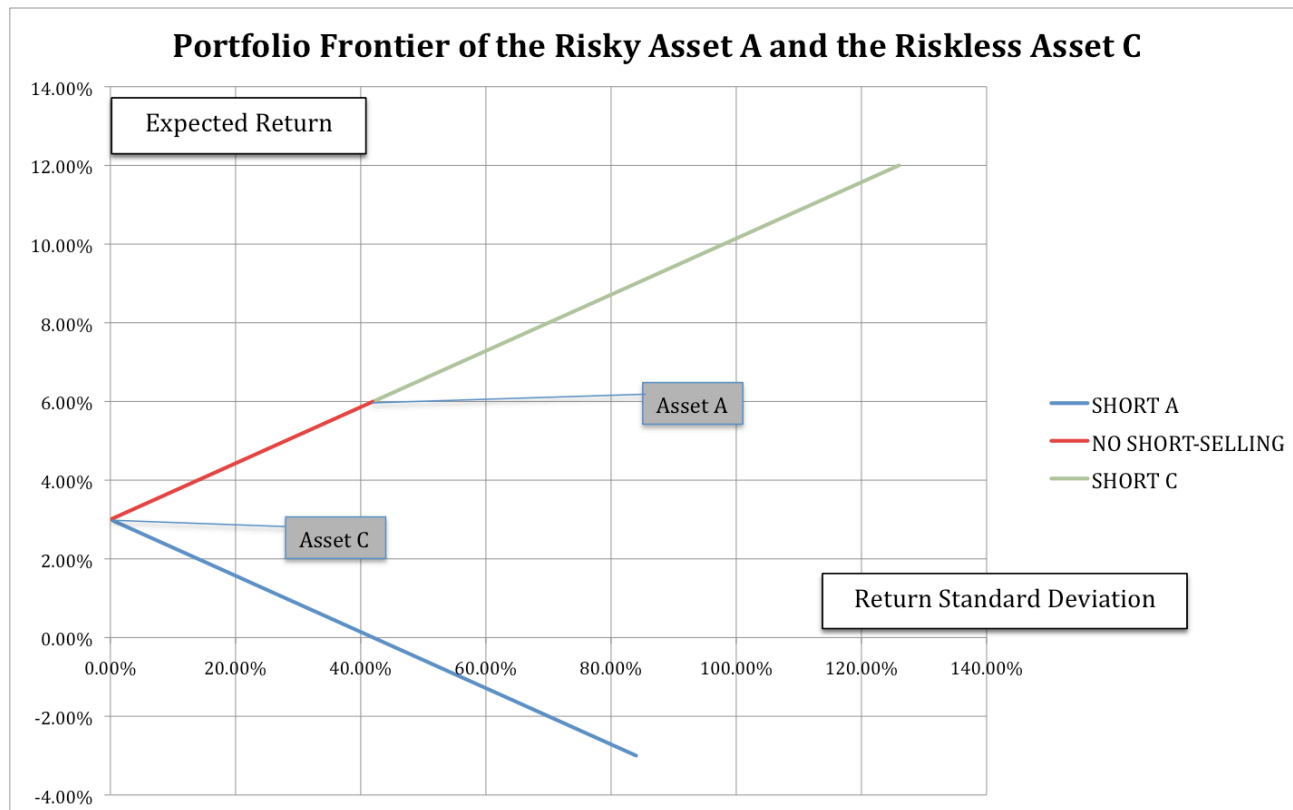
The resulting return standard deviation is

$$\text{Std}(-0.5 \cdot r_A + 1.5 \cdot r_B) = \sqrt{\text{Var}[-0.5 \cdot r_A + 1.5 \cdot r_B]} = \sqrt{0.1198} \approx 0.3461 = 34.61\%.$$

It follows that the standard deviation in \$ is

$$\$50,000 \cdot 34.61\% = \$17,305.$$

(e)



(f) The standard deviation of a portfolio that puts weight  $v$  on asset A and weight  $1-v$  on asset C satisfies

$$Std(v \cdot r_A + (1-v) \cdot r_C) = v \cdot Std(r_A).$$

Therefore, the desired portfolio weight  $v$  satisfies

$$0.63 = v \cdot Std(r_A) \Leftrightarrow 0.63 = v \cdot 0.42 \Leftrightarrow v = 1.5.$$

(g) The expected return of portfolios with weight  $v$  on asset A and weight  $1-v$  on asset C satisfies

$$E[v \cdot r_A + (1-v) \cdot r_C] = v \cdot E[r_A] + (1-v) \cdot r_C = v \cdot 0.06 + (1-v) \cdot 0.03 = 0.03 + 0.03 \cdot v.$$

Therefore, the range of portfolio weights for which the expected return is negative is given by

$$0.03 + 0.03 \cdot v < 0 \Leftrightarrow v < -1.$$