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Econ 252 - Financial Markets

Spring 2011

Professor Robert Shiller

Final Exam – Practice Exam Suggested Solution

Part I.

1. Lecture 22 on "Public and Non-Profit Finance."

With a nonprofit, there is no equity. All profits are held in the company, to be used for the stated purposes of the nonprofit. Nonprofits do have debt. Nonprofits do pay employee bonuses (42% did so in 2007-8). Nonprofits have to compete with forprofits in the market for the services they sell and in the labor market for the employees they hire, and so there are incentives that push them together to behave similarly to each other.

2. Lecture 21 on "Exchanges, Brokers, Dealers, Clearinghouses."

She is a dealer, since she is charging no commission, but is profiting from the spread between the price she paid and the price she sold to you, rather than being paid a commission for helping you buy from the third party.

3. Fabozzi et al., p. 257.

Expected excess return is equal to the expected excess return on each of a number of factor portfolios, each multiplied by its own factor beta.

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4. Lecture 20 on "Professional Money Managers and their Influence."

A trust company holds assets for another party and manages and distributes them according to the instructions left by that party. Trusts will survive the death of the party.

5. Guest lecture by Laura Cha.

She said regulation is well-developed in China today, but enforcement is inconsistent. China is developing so fast that it cannot keep up with enforcing all the regulations. But the regulation is better than that of the U.S. not long ago. China is the largest recipient of foreign direct investment and so it must be that foreigners are trusting of Chinese regulations.

6. Lecture 13 on "Banks."

The Panic of 2007-2009 was essentially a run on the sale and repurchase market (the "repo" market), which is a very large, short-term market that provides collateralized financing for a large range of securitized products. Repurchase agreements are economically like demand deposits; they play the same role as demand deposits, but for firms operating in the capital markets.

7. Fabozzi et al., p. 83.

Operating targets are monetary and financial variables (such as federal funds rate or exchange rate) whose changes tend to bring about changes in intermediate targets, which may include long-term interest rates or monetary aggregates.

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8. Lecture 18 on "Monetary Policy."

Reserve Requirements are related to the shortest run problems, runs on banks, for which banks need vault cash or an account at the central bank. Capital requirements are more long-term, about the banks' assets declining in value. Reserve requirements against saving account (time deposit) balances are only 1%. Reserve requirements against checking accounts can be 10%. Some governments do not impose reserve requirements. Basel III imposes capital requirements on banks of all G-20 countries.

9. Lecture 13 on "Banks."

Rating agencies give letter grades (such as AAA for the best) to securities regarding their probability of default. Rating agencies have been criticized for allowing themselves to be influenced, in the awarding of these grades, by fees paid to them by issuers, and giving high ratings to securities that later defaulted.

10. Lecture 11 on "Behavioral Finance and the Role of Psychology."

Adam Smith, people have a desire for praise. But they don't enjoy being praised for something they did not do. As people mature, if they mature successfully, the desire for praise morphs into a desire for praiseworthiness. The financial world functions as well as it does because there is this desire.

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11. Shiller manuscript, chapter 8.

Claire Hill, in her article "Securitization: A Low-Cost Sweetener for Lemons," offered as a reason that the securitization market exists that it helps solve the "lemons problem" defined by George Akerlof, that there is no market for (in Gorton's words) "informationally sensitive" securities. Investors do not want to do research to find out. Dividing up the evaluation of a company's securities into pieces that allow specialized evaluators to do their job lowers the information cost of all of its securities.

12. Fabozzi et al, p. 295.

Coupon is fixed announced percent of principal at maturity, principal is adjusted semiannually for CPI inflation. If there is deflation, coupons may fall below their initial value, but the terminal payment has a floor equal to the original principal.

13. Lecture 19 on "Investment Banks."

Ellis was struck by the singular focus on making money, no interest in making friends unless they were influential, no interest in being in the public eye. He quotes Whitehead stating the following principles:

- Our client's interests always come first.
- Our assets are people, capital and reputation.
- Uncompromising determination to achieve excellence in everything we undertake.
- We stress creativity and imagination in everything we do.
- The boss usually decides—not the assistant treasurer. Do you know the boss?
- You never learn anything when you're talking.
- The respect of one person is worth more than acquaintance with 100.
- There's nothing worse than an unhappy client.

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14. Fabozzi et al., p. 126.

Load funds make a sales charge, often a substantial up-front charge that reduces the net asset value as soon as the investment is made. In the U.S., the Financial Industry Regulatory Authority (FINRA) allows as much as 8.5% initial charges. Despite negative publicity, load funds are prospering, since advisers have more of an incentive to push them.

15. Fabozzi et al., p. 326.

The government might lower high-income marginal tax rates or end the taxexemption. News stories that they *might* do this could cause the value of long-term municipal bonds to drop, even if the government has not done so.

16. Fabozzi et al., p. 279.

The rule eliminates the two-year holding period requirement for privately placed securities for large institutions. Individual investors are still subject to the two-year holding requirement.

17. Assigned reading *Wall Street and the Country: A Study of Recent Financial Tendencies* by Charles Conant, pp. 94-95.

"It is true, no doubt, that the stock market sometimes seems unduly sensitive to these widely separated and isolated events, but if one looks to the fundamental conditions which govern economic society, it must be clear that it is better that it should be too sensitive than not sufficiently so. It is better that any rumor of war, with a threatened cessation of production and consumption, should be reflected on organized markets than that people should go on recklessly investing capital in enterprises which may afterwards prove unproductive."

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18. Lecture 4 on "Portfolio Diversification and Supporting Financial Institutions."

The equity premium is normally positive, so one would never do so, since it would create a portfolio, which has lower expected return than the riskless asset and would have positive variance and hence would be dominated by the riskless asset.

19. Fabozzi et al., p. 438.

A financial asset issued by a bank or thrift that indicates a specified sum of money has been deposited. If nonnegotiable, the buyer must wait until the maturity date, if negotiable, they may sell the CD in the market.

20. Fabozzi et al., p. 523; Lecture 15 on "Forward and Futures Markets."

A forward market is over the counter and between two parties, suffers from counterparty risk but can be specially tailored to the clients' needs. A futures market is on an exchange, is standardized, and has no counterparty risk. It cannot be tailored to clients' needs, but, since it is standardized, it offers better price discovery.

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Part IB.

Question 1

(a) The annual returns of the market index are as follows:

$$2005 \rightarrow 2006: \frac{530-500}{500} = 0.06 = 6\%, 2006 \rightarrow 2007: \frac{530-530}{530} = 0 = 0\%,$$

$$2007 \rightarrow 2008: \frac{519.4-530}{530} = -0.02 = -2\%, 2008 \rightarrow 2009: \frac{555.76-519.4}{519.4} \approx 0.07 = 7\%,$$

$$2009 \rightarrow 2010: \frac{577.99-555.76}{555.76} \approx 0.04 = 4\%.$$

Therefore, the expected return of the market index is

$$E[r_M] = \frac{1}{5}(0.06 + 0 - 0.02 + 0.07 + 0.04) = 0.03 = 3\%.$$

(b) The CAPM implies that

$$E[r_A] - r_f = \beta_A(E[r_M] - r_f) \Leftrightarrow E[r_A] = r_f + \beta_A(E[r_M] - r_f).$$

It follows that the expected return of asset A is

$$E[r_A] = r_f + \beta_A (E[r_M] - r_f) = 0.012 + 0.7 \cdot (0.03 - 0.012) = 0.0246 = 2.46\%.$$

(c) The CAPM implies that

$$E[r_A] - r_f = \beta_{new}(E[r_M] - r_f) \Leftrightarrow E[r_A] = r_f + \beta_{new}(E[r_M] - r_f).$$

It follows that the new expected return of asset A is

 $E[r_A] = r_f + \beta_{new}(E[r_M] - r_f) = 0.012 + 0.2 \cdot (0.03 - 0.012) = 0.0156 = 1.56\%.$

If the β of asset A changes from 0.7 to 0.2, the expected return of asset A decreases from 2.46% to 1.56%. Because the variance of asset A is assumed to be unchanged, the change in β implies that asset A covaries less with the market in the β =0.2 scenario. In consequence, asset A offers bigger diversification benefits. In consequence, the expected return that asset A needs to offer in order to induce market participants to hold it decreases.

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Question 2

As stated, a time period refers to six months.

(a) It is possible to compute the 30-month spot rate.

Denote the one-period quantity of the desired 30-month spot rate by z.

The 12-month spot rate for one period is $0.5 \cdot 1.5\% = 0.75\% = 0.0075$. The forward rate between 12 and 30 months for one period is $0.5 \cdot 2.4\% = 1.2\% = 0.012$.

12 months correspond to 2 periods and 30 months to 5 periods. In consequence, there are 3 period between 12 and 30 months.

One therefore obtains that z satisfies the following identity:

$$(1.012)^3 = \frac{(1+z)^5}{(1.0075)^2}.$$

This implies that $z\approx 1.02\%$. That is, the annualized 30-month spot rate equals approximately 2.04%.

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(b) As in part (a), the 12-month spot rate for one period is 0.5•1.5%=0.75%=0.0075. Moreover, the 30-month spot rate for one period has been computed as 0.5•2.04%=1.02%=0.0102.

The investment strategy is as follows:

- Borrow \$a/(1.0075)² today at the 12-month spot rate, to be repaid 12 months from now.
- Invest the borrowed money at the 30-month spot rate for 30 months.

The balance of this investment strategy evolves as follows:

• Today:

You invest all of the borrowed money. That is, you do not need to put up any of your own capital and there is no capital left over at the end of today.

• 12 months from now:

The 12-month loan matures. So, you need to repay the originally borrowed amount multiplied by the 12-month spot rate for three periods, that is,

$$\frac{a}{(1.0075)^2} \cdot (1.0075)^2 = a.$$

You use the \$a that you collect 12 months from now to repay the loan.

In summary, you start with \$a and you have no capital left at the end of 12 months from now.

• 30 months from now:

The 30-month investment matures. So, you collect the invested amount multiplied by the fifth power of the 30-month spot rate for one period. The fifth power originates from the fact that five periods pass between today and 30 months from now. That is,

$$\frac{a}{(1.0075)^2} \cdot (1.0102)^5 = a \cdot \frac{(1.0102)^5}{(1.0075)^2}.$$

So, you start with no money at all and you have $a \cdot (1.0102)^5/(1.0075)^2$ at the end of 30 months from now.

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In summary, the investment strategy turns \$a into \$a • $(1.0102)^5/(1.0075)^2$. The factor $(1.0102)^5/(1.0075)^2$ exactly corresponds to the factor $(1.012)^3$, which gives rise to an annualized forward rate between 12 and 30 months of 2.4%.

(c) The Pure Expectation Theory states that forward rates exclusively represent the expected future spot rates.

The expected 18-month spot rate 12 months from now is equal to the forward rate between 12 and 30 months (that is, between 12 months and 18 months later than that). Therefore, the expected 18-month spot rate 12 months from now is 2.4% according to the Pure Expectations Theory.

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Question 3

(a) The dividend payment of \$8 for asset X translates into a dividend rate of 4%, as 8/200=0.04.

The fair value of the described futures contract is given by the expression

(1+r-y)•spot price of asset X,

where r is the riskless interest rate and y is the dividend rate for asset X.

In consequence, the fair value of the described futures contract is

 $(1+0.05-0.04) \cdot 200=202.$

Therefore, when the actual futures price is \$210, an investor can make a riskless profit without using any of his own capital with a "cash and carry trading strategy":

- Period 0:
 - 1. Sell futures contract.
 - 2. Borrow \$200.
 - 3. Use borrowed money to buy underlying asset for \$200.
- Period 1:
 - 1. Collect \$8 because of the X's dividend payment.
 - 2. Use underlying asset to settle futures contract, receive \$210.
 - 3. Pay off loan with \$210=1.05•\$200.

The total arbitrage profit from these transactions is \$8.

(b) As computed in part (a), the fair value of the described futures contract is \$202. As the actual futures price is below the fair value price, one would need to implement the "reverse cash and carry trading strategy" in order to exploit this arbitrage opportunity. However, this strategy entails short-selling asset X, which is prohibited. Therefore, it is not possible to make an arbitrage profit.

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Question 4

(a) As the market portfolio is always located on the Tangency Line, its Sharpe ratio equals 0.12. Given that the expected return of the market portfolio equals 5%, the return standard deviation of the market portfolio satisfies

$$0.12 = \frac{0.05 - 0.02}{\sigma_{M}} \Leftrightarrow \sigma_{M} = \frac{0.05 - 0.02}{0.12} = 0.25 = 25\%.$$

(b) The Mutual Fund Theorem states that, in mean-variance portfolio-analysis involving a risk-free asset, any investor with the standard μ - σ preferences will optimally choose a portfolio consisting of the risk-free asset and of the tangency portfolio. Importantly, the weight on the tangency portfolio will be non-negative (if the equity premium is positive, which it generally is). That is, each investor with the standard μ - σ preferences only holds one risky asset, the tangency portfolio, in his portfolio.

However, the Mutual Fund Theorem does not prescribe the exact portfolio that an investor holds. The only valid conclusion is that the optimal portfolio will be located on the Tangency Line.

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(c) Portfolio A has the following expected return:

$$E[r_A] = 0.06 \cdot E[r_M] + 0.94 \cdot r_f = 0.0218 = 2.18\%.$$

Moreover, as one of its assets is riskless, portfolio A's standard deviation is equal to

$$\sigma[r_A] = 0.06 \cdot \sigma[r_M] = 0.015 = 1.5\%.$$

Portfolio B has the following expected return:

$$E[r_A] = 0.72 \cdot E[r_M] + 0.28 \cdot r_f = 0.0416 = 4.16\%.$$

Moreover, as one of its assets is riskless, portfolio A's standard deviation is equal to

 $\sigma[r_A] = 0.72 \cdot \sigma[r_M] = 0.18 = 18\%.$

It follows that the four relevant portfolios generate the following utility for the agent:

- risk-free portfolio: utility 0.02 4•0²=0.02,
- market portfolio: utility 0.05 4•0.25²=-0.2,
- portfolio A: utility 0.0218 4•0.015²=0.0209,
- portfolio B: utility 0.0416 4•0.18²=-0.088.

So, the agent optimally chooses portfolio A.

If one uses the return standard deviation of 30% for the market portfolio, one obtains a utility value of $0.05-4 \cdot 0.3^2$ =-0.31 for the market portfolio, which does not change the optimality of portfolio A for the agent.

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Question 5

Underlying	S _T ≤20	S _T >20
Payoff	20-S _T	0
Put P ₁ with E=20		
Payoff	ST	ST
Stock S _T		
Payoff Portfolio	20	ST
$P_1 + S_T$		

(a) The payoff of the described portfolio at maturity is obtained as follows:

The portfolio P_1+S_T costs \$10+\$25=\$35. It follows that the profit of the portfolio at maturity is as follows:

Underlying	S _T ≤20	S _T >20
Profit Portfolio	-15	S _T -35
P_1+S_T		



An investor might want to construct this portfolio to set up a stock position with limited maximum loss.

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(b) The payoff of the described portfolio at maturity is obtained as follows:

Underlying	S _T ≤15	15 <s<sub>T≤20</s<sub>	S _T >20
Payoff	20-S _T	20-S _T	0
Put P ₁ with E=20			
Payoff	15-S _T	0	0
Put P ₂ with E=15			
Payoff Portfolio	5	20-S _T	0
P ₁ -P ₂			

The portfolio P_1 - P_2 costs \$10-\$8=\$2. It follows that the profit of the portfolio at maturity is as follows:

Underlying	S _T ≤15	15 <s<sub>T≤20</s<sub>	S _T >20
Profit Portfolio P ₁ -P ₂	3	18-S _T	-2



An investor might want to construct this portfolio if he thinks that the price of the stock will go down moderately between the time of construction and the maturity date. Moreover, he puts a limit on his losses if the stock increases.

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Question 6

The price of the underlying XYZ evolves as follows:

• A year from now:

$$S(u) = u \cdot S(0) = 180, S(d) = d \cdot S(0) = 60.$$

• Two years from now:

$$S(uu) = u^2 \cdot S(0) = 270, S(ud) = S(du) = u \cdot d \cdot S(0) = 90, S(dd) = d^2 \cdot S(0) = 30.$$

(a) The one-period Binomial Asset Pricing Model has the following schematic form:



The relevant quantity is the one-period hedge ratio for the call option C₁.

$$H = \frac{C_1(u) - C_1(d)}{(u - d)S(0)} \Leftrightarrow H = \frac{\max[180 - 60, 0] - \max[60 - 60, 0]}{(1.5 - 0.5) \cdot 120} = 1.$$

(b) The desired quantity is $C_1(0)$, which satisfies the following identity:

$$\frac{H \cdot S(u) - C_1(u)}{H \cdot S(0) - C_1(0)} = 1 + r \Leftrightarrow \frac{1 \cdot 180 - \max[180 - 60, 0]}{1 \cdot 120 - C_1(0)} = 1.05$$
$$\Leftrightarrow C_1(0) \approx 62.85.$$

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(c) The two-period Binomial Asset Pricing Model has the following schematic form:



The desired quantity $C_2(0)$ will be obtained via backward induction.

At the upper node in period 1, that is, after the stock price increases once:

$$H(u) = \frac{C_2(uu) - C_2(ud)}{(u-d) \cdot S(u)} \Leftrightarrow H(u) = \frac{\max[270 - 60, 0] - \max[90 - 60, 0]}{(1.5 - 0.5) \cdot 180} = 1$$

Furthermore, the price of C_2 at the upper node in period 1 satisfies the following identity:

$$\frac{H \cdot S(uu) - C_2(uu)}{H \cdot S(u) - C_2(u)} = 1 + r \Leftrightarrow \frac{1 \cdot 270 - \max[270 - 60, 0]}{1 \cdot 180 - C_2(u)} = 1.05$$
$$\Leftrightarrow C_2(u) \approx 122.86.$$

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At the bottom node in period 1, that is, after the stock price increases once:

$$H(d) = \frac{C_2(du) - C_2(dd)}{(u-d) \cdot S(d)} \Leftrightarrow H(d) = \frac{\max[90 - 60, 0] - \max[30 - 60, 0]}{(1.5 - 0.5) \cdot 60} = 0.5.$$

Furthermore, the price of C_2 at the bottom node in period 1 satisfies the following identity:

$$\frac{H \cdot S(du) - C_2(du)}{H \cdot S(d) - C_2(d)} = 1 + r \Leftrightarrow \frac{0.5 \cdot 90 - \max[90 - 60, 0]}{0.5 \cdot 60 - C_2(d)} = 1.05$$
$$\Leftrightarrow C_2(d) \approx 15.71.$$

Hence, $C_2(d) \approx 15.71 is the answer to the first question.

Finally, at the initial node:

$$H(0) = \frac{C_2(u) - C_2(d)}{(u-d) \cdot S(0)} \Leftrightarrow H(0) = \frac{122.86 - 15.71}{(1.5 - 0.5) \cdot 120} \approx 0.89.$$

Furthermore, the price of C₂ at the initial node satisfies the following identity:

$$\frac{H \cdot S(u) - C_2(u)}{H \cdot S(0) - C_2(0)} = 1 + r \Leftrightarrow \frac{0.89 \cdot 180 - 122.86}{0.89 \cdot 120 - C_2(0)} = 1.05$$
$$\Leftrightarrow C_2(0) \approx 71.24.$$

Hence, $C_2(0) \approx 71.24 is the answer to the second question.

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(d) Recall that the put-call parity is

$$C + \frac{E}{\left(1+r\right)^T} = S + P,$$

where T denotes time to maturity. It holds at any time period and for any state of the stock price evolution.

Therefore, the price of a 2-year put with the same strike price as C_2 a year from now after the price has gone down once is

$$15.71 + \frac{60}{1 + 0.05} = 60 + P(d) \Leftrightarrow P(d) = 12.85.$$

Analogously, the price of this put option today is:

$$71.24 + \frac{60}{(1+0.05)^2} = 120 + P(0) \Leftrightarrow P(0) = 5.66.$$