Summary & Lessons of the First Class

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The "Grade Game". We played the following game:

> *"Without showing your neighbor what you are doing, write down on a form either the letter* $\boldsymbol{\alpha}$ *or the letter* $\boldsymbol{\beta}$*. Think of this as a 'grade bid'. We will randomly pair your form with one other form. Neither you nor your pair will ever know with whom you were paired. Here is how grades* may *be assigned for this course.*

- *If you put* $\boldsymbol{\alpha}$ *and your pair puts* $\boldsymbol{\beta}$*, then you will get grade A, and your pair grade C.*
- *if both you and your pair put* $\boldsymbol{\alpha}$*, then you both will get grade B-.*
- *if you put* $\boldsymbol{\beta}$ *and your pair puts* $\boldsymbol{\alpha}$*, then you will get grade C, and your pair grade A.*
- *if both you and your pair put* $\boldsymbol{\beta}$*, then you will both get grade B+."*

The possible choices, $\boldsymbol{\alpha}$ or $\boldsymbol{\beta}$, are called 'strategies'. The grades — for example, (A,C) — are 'outcomes'. We can record such information as follows:

Outcome Matrix

		pair $\boldsymbol{\alpha}$	pair $\boldsymbol{\beta}$
me	$\boldsymbol{\alpha}$	B-,B-	A, C
	$\boldsymbol{\beta}$	C, A	B+,B+

What strategy should a rational person choose in the Grade Game? To answer this, we first need to know what that person cares about: what 'payoff' does each outcome yield for this person. Game theory can not tell us what payoffs to assign to outcomes. This depends on the preferences (and moral sentiments?) of the players, not just you but also your opponents. But game theory has a lot to say about how to play the game once payoffs are known.

Possible payoffs: Evil gits. For example, if every player is an "*evil git*", each only caring about her own grade then (assuming she prefers A to B etc.) the payoffs might be as follows:

Payoff Matrix 1

		pair $\boldsymbol{\alpha}$	pair $\boldsymbol{\beta}$
me	$\boldsymbol{\alpha}$	$0, 0$	$3, -1$
	$\boldsymbol{\beta}$	$-1, 3$	$1, 1$

What should I choose in this case? If my pair chooses $\boldsymbol{\alpha}$, then my choosing $\boldsymbol{\alpha}$ yields me 0 whereas my choosing $\boldsymbol{\beta}$ yields me only -1. If my pair chooses $\boldsymbol{\beta}$, then my choosing $\boldsymbol{\alpha}$ yields me 3 whereas my choosing $\boldsymbol{\beta}$ yields me only 1. So, in either case, my choosing $\boldsymbol{\alpha}$ is better. Formally, we say:

Definition. My strategy $\boldsymbol{\alpha}$ *strictly dominates* my strategy $\boldsymbol{\beta}$ if my payoff from $\boldsymbol{\alpha}$ is strictly higher than that from $\boldsymbol{\beta}$ **regardless of others' choices**.

[**Extra Definition.** My strategy $\boldsymbol{\alpha}$ *weakly dominates* my strategy $\boldsymbol{\beta}$ if my payoff from $\boldsymbol{\alpha}$ is as high as that from $\boldsymbol{\beta}$ regardless of others' choices, and is strictly higher for at least one such choice.]

Lesson 1. *You should never play a strictly dominated strategy.*

Unfortunately, the reasoning is the same for my pair: given these payoffs, she will also choose $\boldsymbol{\alpha}$. We will end up both getting B- even though there is a possible outcome (B+,B+) that is better for both of us. To use some economics jargon: the outcome (B-,B-) is *Pareto inefficient*.

Lesson 2: *Rational play by rational players can lead to bad outcomes*

Games like this one are called *Prisoners' Dilemmas*. Other examples of prisoners' dilemmas include: "Law & Order" episodes; price wars; (lack of) room tidying by Yale undergrads;.... Remedies include: contracts enforced by the courts or by the Mafia; repeated play (we will return to this later in the semester).

Other possible payoffs for the grade game: indignant angels. In contrast to the case where all players are evil, suppose that each person cares not only about her own grade but also about the grade of the person with whom she is paired. For example, each player might be an "*indignant angel*": she likes getting an A but she feels guilty that this is at the expense of her pair getting a C. The guilt lowers her payoff from 3 to -1. Conversely, if she gets a C because her pair gets an A, indignation reduces the payoff from -1 to -3. In this case, payoffs would be:

Payoff Matrix 2

		pair $\boldsymbol{\alpha}$	pair $\boldsymbol{\beta}$
me	$\boldsymbol{\alpha}$	0, 0	$-1, -3$
	$\boldsymbol{\beta}$	$-3, -1$	1, 1

What should I choose in this case? As before, if my pair chooses $\boldsymbol{\alpha}$, then my choosing $\boldsymbol{\alpha}$ yields a higher payoff than my choosing $\boldsymbol{\beta}$. If my pair chooses $\boldsymbol{\beta}$, however, then my choosing $\boldsymbol{\beta}$ yields a higher payoff than my choosing $\boldsymbol{\alpha}$. In this case, no strategy is dominated. The best choice depends on what I think my pair is likely to do. Later in the course, we will examine games like this called "co-ordination games".

Lesson 3: *To figure out what actions you should choose in a game, a good first step is to figure out what are your payoffs (what do you care about) and what are other players' payoffs.*

The Evil Git versus the Indignant Angel: What if I am an evil git but I know my opponent is an indignant angel? In this case the payoffs would be:

Payoff Matrix 3

		pair $\boldsymbol{\alpha}$	pair $\boldsymbol{\beta}$
me	$\boldsymbol{\alpha}$	$0, 0$	$3, -3$
	$\boldsymbol{\beta}$	$-1, -1$	$1, 1$

My strategy α strictly dominates my strategy β. Check this. Therefore I should choose α.

The Indignant Angel versus the Evil Git: What if I am an indignant angel but I know my opponent is an evil git? In this case the payoffs would be:

Payoff Matrix 4

		pair $\boldsymbol{\alpha}$	pair $\boldsymbol{\beta}$
me	$\boldsymbol{\alpha}$	$0, 0$	$-1, -1$
	$\boldsymbol{\beta}$	$-3, 3$	$1, 1$

Neither of my strategies dominates the other. Check this. But, my pair's strategy α strictly dominates her strategy β. Therefore, if I know she is rational then I know she'll play α. In which case, I should play α (to get $0 > -3$).

Lesson 4. *If you do not have a dominated strategy, put yourself in your opponents' shoes to try to predict what they will do. For example, in their shoes, you would not choose a dominated strategy.*

What if I do not know the payoffs of the person I am playing against? If I am an evil git, this is easy: my strategy α is strictly dominant so I should choose α. It does not matter whom I am playing. But if I am an indignant angel, the issue is more delicate. It depends (among other things) on what proportion of gits and angels I think there are in the population. With luck, we will get to games like these (sometimes called Bayesian games) towards the very end of the semester.

What do real people do in Prisoners' Dilemmas? Only about 15% of the class chose $\boldsymbol{\beta}$ in the grade game. In larger experiments with 'normal people', about 30% chose (the analogue of) $\boldsymbol{\beta}$. Does this mean that Yale students are smarter than normal folk? Not necessarily. It could just be:

Lesson 5. *Yale students are evil.*